The Stagnation Point in Free Coating

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Groenveld (1970a) has predicted that the location of the stagnation point (on the upper portion of a free coating film) occurs where the film thickness h is three times the thickness h_0 in the constant thickness region. In other words, the Groenveld stagnation thickness h_{ST}^* is given by

$$h_{ST}^* = 3\, h_0 \tag{1}$$

Equation (1) was derived by assuming that the velocity profile in the constant-thickness region as approximate plug flow, which is typical of low speeds and capillary numbers below 10^{-2}. Here capillary number $Ca \equiv U_w\,(\mu/\sigma)$.

The purpose of this note is to derive a more general prediction of the stagnation thickness h_{ST}—one which does not place any restrictions on the shape of velocity profile in the constant thickness region. It will be shown that the result predicts an influence of capillary number.

One experimental method of obtaining free coating is shown in Figure 1 where point B is the stagnation point of interest here. The prediction developed in this note, however, is not limited to this geometry; the prediction is developed for an infinite liquid bath and applies to other geometries, including the rotating disk studied by Groenveld (1970a).

The bases for the prediction are the well-known equations of Landau-Levich (1942), which have been summarized by Levich (1962), White and Tallmadge (1965) and others. These equations were developed for the flatter (upper) portion of the free coating film and are noted briefly below.

BASIC EQUATIONS

Approximating the flow in the film as quasi-one-dimensional and neglecting the inertial terms, the x component of the equation of motion is written as

$$\mu \frac{d^2u}{dy^2} - \rho g - \frac{\partial P}{\partial x} = 0$$

or

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{\partial P}{\partial x} + \frac{\rho g}{\mu} \equiv A \tag{2}$$

Here the y axis is perpendicular to the plate and the x axis vertical upward. The boundary conditions for the film are:

1. No slip at the wall.
2. Negligible tangential stress at the liquid-gas interface.
3. Flux Q_0 in the top, (the constant thickness region of h_0) known in terms of h_0.
4. Normal stress balance across the liquid-gas interface.

Integrating the equation of motion twice at constant height and evaluating the two integration constants by the no slip and tangential stress boundary conditions (Numbers 1 and 2) yields the velocity profile

$$u = \frac{1}{2}Ay^2 - Ahy + U_w \tag{3}$$

From the definition $Q \equiv \int_0^h u\,dy$, the flux Q_h in the film region is given by $Q_h = U_w h - (Ah^3/3)$. Here $Q_0 = U_w h - (\rho g h_0^3/3\mu)$. Equating the film flux Q_h with the known flux Q_0 (of boundary condition 3) by continuity yields the relationship between h and h_0

$$U_w h_0 - \frac{\rho g h_0^3}{3\mu} = U_w h - \frac{Ah^3}{3}$$

or

$$U_w h_0 - \frac{\rho g h_0^3}{3\mu} = U_w h - \frac{h^3}{3}\left(\frac{1}{\mu}\frac{\partial P}{\partial x} + \frac{\rho g}{\mu}\right) \tag{4}$$

In the previous work, the pressure term was replaced using an approximation for the fourth boundary condition, namely that $\partial P/\partial x \cong \sigma d^3h/dx^3$. Thus Equation (4) is in a more general form than that given earlier. The reason for showing the pressure term without simplification is to emphasize that the derivation below holds for a wider range of $\partial P/\partial x$ assumptions.

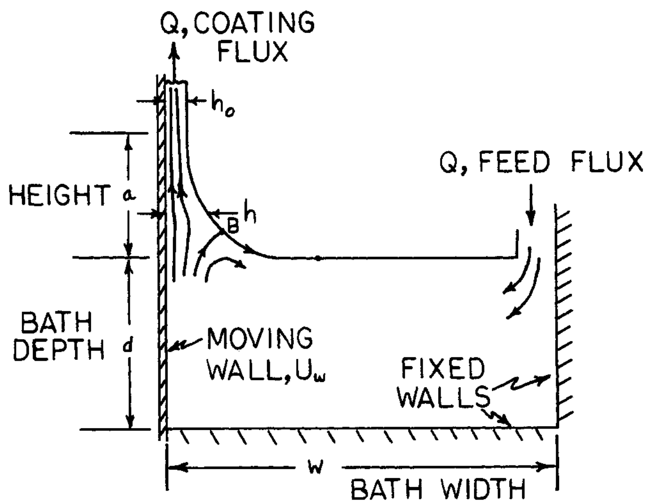

Fig. 1. Sketch of one geometry used for free coating.

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SURFACE VELOCITY IN THE FILM, U_s

The interfacial velocity in the film is determined from the velocity profile, Equation (3), by simple substitution of $u = U_s$ at $y = h$. Thus

$$U_s = U_w - \frac{Ah^2}{2} = U_w - \frac{h^2}{2}\left(\frac{1}{\mu}\frac{dP}{dx} + \frac{\rho g}{\mu}\right) \quad (5)$$

To obtain a more useful description of surface velocity, we note that Equations (4) and (5) may be taken simultaneously to eliminate the pressure term. An equivalent procedure is to eliminate the A parameter, which is given by Equation (4) as

$$Ah^3 = 3U_w (h - h_0) + h_0^3 (\rho g/\mu) \quad (4a)$$

Thus substituting film Equation (4) into velocity Equation (5) leads to

$$\frac{U_s}{U_w} = 1 - \frac{1}{2h}\left[3(h - h_0) + \frac{\rho g h_0^3}{\mu U_w}\right] \quad (6)$$

Equation (6) is apparently a new expression for surface velocity. The alternative expression, in terms of the nondimensional film thickness, $T_0 \equiv h_0 (\rho g/\mu U_w)^{1/2}$, is given by

$$\frac{U_s}{U_w} = 1 - \frac{1}{2h}[3(h - h_0) + h_0 T_0^2] \quad (6a)$$

STAGNATION POINT h_{ST}

Using surface velocity Equation (6a), the location of the stagnation point, where U_{ST} is taken as zero, is predicted to be

$$0 = 2h_{ST} - 3(h_{ST} - h_0) - h_0 T_0^2$$

or

$$\frac{h_{ST}}{h_0} = 3 - T_0^2 \quad (7)$$

Equation (7) is the desired prediction of stagnation point location for a range of Ca. The relation between thickness h_0 (or T_0) and Ca has been reported by several authors, including Landau-Levich (1942), Van Rossum (1958), White and Tallmadge (1965), Groenveld (1970a), and others.

Equation (7) suggests that the stagnation point h_{ST} varies from $3h_0$ at very low Ca (where $T_0 \to 0$) to something more than 2 h_0 at large Ca (since $T_0 \leq 1$) and is about 2.5 h_0 at Ca near 1 (where $T_0 \sim 0.7$).

Assumption of plug flow in the h_0 region is equivalent to dropping one term in Equation (4) to obtain

$$U_w h_0 = U_w h - (Ah^3/3) \quad (8)$$

Substitution of A from film Equation (8) into the velocity Equation (5) leads, as expected, to the simplified prediction of $h_{ST}/h_0 = 3$, given by Equation (1).

DISCUSSION

It is difficult to obtain precise data on stagnation points in terms of h_0. The few data which are available are also limited in speed to Ca near 10°. For example, Groenveld (1970b) reports disk data of 2.8 ± 0.3 h_0, using glycerol with viscosity of 1.00 Ns/m² at speeds such that Ca is probably between 10^{-1} and 10^{+1}, and the authors have observed flat sheet data in the 2.5 to 3.0 h_0 range. It appears that the data agree with the prediction given above within about 0.3 h_0 so that the order of magnitude seems to be correct. Furthermore, studies with two-dimensional flow predictions (Lee and Tallmadge, 1972a) indicate values of 2.3 to 2.4 h_0 for the Ca range of 2 to 24, which is also excellent agreement with the prediction given above.

Experimental evidence has shown, for a given fluid and geometry, that the stagnation point disappears in shallow baths. It is believed that this disappearance phenomena would be influenced by geometry and, to some lesser extent, by fluid properties.

In view of the assumption of quasi-one-dimensional flow, Equation (7) appears to be surprisedly precise. It appears to be the first prediction of the influence of capillary number on the stagnation point location h_{ST}. It was presented earlier (Lee and Tallmadge, 1972b) without proof or discussion. Knowledge of the stagnation point is useful for characterizing flow in several ways, such as determining where surface downflow begins and providing a delineation between the upper and lower regions of the film.

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NOTATION

A = parameter, Equation (2)
Ca = capillary number, $Ca = U_w \mu/\sigma$
g = acceleration of gravity
h = meniscus thickness at any point, mm
h_0 = film thickness, constant thickness region, mm
h_{ST} = thickness at the (upper) stagnation point, mm
h_{ST}^* = estimate of h_{ST}, Equation (1)
P = pressure, N/m²
Q = flux, mass flow rate per unit width, g/mm
Q_h = flux in the variable h region, g/mm
Q_0 = flux in the constant thickness region, g/mm
T_0 = film thickness, dimensionless, $h_0 (\rho g/\mu U_w)^{0.5}$
u = vertical velocity, mm/s
U_s = velocity of surface, mm/s
U_{ST} = surface velocity at the stagnation point, mm/s
U_w = coating velocity of belt, mm/s
x = vertical coordinate, meniscus height above liquid level, mm
y = horizontal distance from the belt, mm
μ = liquid viscosity, N-s/m²
ρ = liquid density, g/m³
σ = surface tension of the liquid-air interface, N/m

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